

NASA TECHNICAL NOTE



NASA TN D-8362 *C.1*

NASA TN D-8362

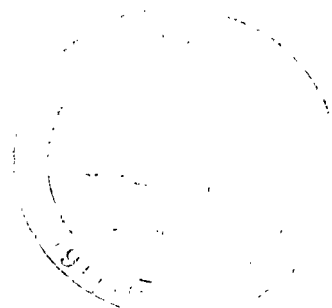
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# FATIGUE LIFE ANALYSIS FOR TRACTION DRIVES WITH APPLICATION TO A TOROIDAL TYPE GEOMETRY

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • DECEMBER 1976



0134112

1. Report No. NASA TN D-8362	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle FATIGUE LIFE ANALYSIS FOR TRACTION DRIVES WITH APPLICATION TO A TOROIDAL TYPE GEOMETRY		5. Report Date December 1976	
7. Author(s) John J. Coy, Stuart H. Loewenthal, and Erwin V. Zaretsky		6. Performing Organization Code	
9. Performing Organization Name and Address NASA Lewis Research Center and U.S. Army Air Mobility R&D Laboratory Cleveland, Ohio 44135		8. Performing Organization Report No. E-8737	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		10. Work Unit No. 505-04	
15. Supplementary Notes		11. Contract or Grant No.	
16. Abstract A contact fatigue life analysis for traction drives has been developed which was based on a modified Lundberg-Palmgren theory. The analysis was used to predict life for a cone-roller toroidal traction drive. A 90-percent probability of survival was assumed for the calculated life. Parametric results were presented for life and Hertz contact stress as a function of load, drive ratio, and size. A design study was also performed. The results were compared to previously published work for the dual cavity toroidal drive as applied to a typical compact passenger vehicle drive train. For a representative duty cycle condition wherein the engine delivers 29 horsepower at 2000 rpm with the vehicle moving at 48.3 km/hr (30 mph) the drive life was calculated to be 19 200 km (11 900 miles).		13. Type of Report and Period Covered Technical Note	
17. Key Words (Suggested by Author(s)) Traction drives; Transmissions; Reliability; Fatigue life; Automotive, Gears; Stress analysis; Lubrication; Friction		14. Sponsoring Agency Code	
18. Distribution Statement Unclassified - unlimited STAR Category 37			
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 32	22. Price* \$4.00

# FATIGUE LIFE ANALYSIS FOR TRACTION DRIVES WITH APPLICATION TO A TOROIDAL TYPE GEOMETRY

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## SUMMARY

A contact fatigue life analysis for traction drives has been developed based on a modified Lundberg-Palmgren fatigue theory which is the accepted basis of rating rolling-element bearing life in the bearing industry. The analysis was used to predict the service life of a modern cone-roller toroidal traction drive. Material and design life adjustment factors were considered. A constant operational traction coefficient of 0.06 was chosen. A minimum contact overload factor of 1.2 was selected to preclude roller slip. An elastohydrodynamic film thickness calculation was performed for the input and output contacts at a representative duty cycle operating point. Life adjustment factors due to advances in materials and lubricant technology were considered.

The traction drive geometry consisted of a double roller per cavity dual cavity toroidal drive with a torus diameter of 11.43 centimeters (4.50 in.), a cavity diameter of 8.84 centimeters (3.48 in.) and a roller crown radius of 3.12 centimeters (1.23 in.). Parametric results were presented for life and Hertz contact stress as functions of load, drive ratio, and size.

To determine vehicle service life, the toroidal drive was incorporated into a typical compact passenger vehicle drive train using a representative duty cycle condition of 29 engine horsepower, 48.3 kilometers per hour (30 mph) vehicle speed, and a drive ratio of 1.2. The mileage life was calculated at 19 200 kilometers (11 900 miles). This is 38 times less than previously stated in the literature for the same condition. At this nominal operating condition the minimum lubricant film thickness to composite surface roughness ratio was calculated as 0.56 on the output contact. The film thickness is marginal to prevent severe roller surface distress.

At severe operating conditions (high drive reduction ratios and high transmitted loads), the brinelling limit for the input disk-roller contact is approached. Shock or dynamic overload protection is marginal.

The present fatigue analysis has shown that life is proportional to the 8.4 power of drive size. To provide for 160 900 kilometers (100 000 miles) of vehicle service at the aforementioned duty cycle point, the cone roller toroidal drive size must be increased to 1.3 times its current size.

## INTRODUCTION

With the increased concern for the nation's dwindling energy resources, the passenger automobile has become a prime target for energy conservation efforts. As an example, in 1970 the transportation sector accounted for more than 24 percent of the total energy consumed in the U.S. and the gasoline consumption of private motor vehicles represents about 55 percent of this portion (ref. 1).

Of the several areas where improvements in vehicle fuel economy can be realized, altering the transmission system is one of the most rewarding (ref. 2). Several independent studies have estimated that the incorporation of a continuously variable transmission (CVT) in place of the conventional automatic transmission in a standard piston engine vehicle would result in a fuel mileage gain of up to 28 percent over the federal driving cycle (refs. 2 and 3). Furthermore, the use of a CVT in conjunction with other vehicle and engine improvements for future vehicles could mean as much as a 60 percent fuel savings (ref. 2). These savings are attributed to two benefits of the CVT. With proper controls the CVT is able to regulate the engine speed to operate along a minimum fuel consumption line. Thus, fuel economy is improved. Also, since better vehicle acceleration is obtained with the CVT through engine speed control, smaller engines may be used adding to the fuel economy.

The goal of developing a commercially viable automotive CVT has brought about renewed interest in traction type transmissions. These transmissions transmit power through shear of the lubricant film at the surface of smooth roller elements which are in highly loaded contact. Speed ratio changes are accomplished by the repositioning of some of the drive rollers which in turn alters the contact rolling radii of the components in the system. The principal advantages that traction CVT's hold over hydraulic types is in their ability to transmit power smoothly and quietly with high efficiency, normally greater than 90 percent.

The automotive traction transmission made its first commercial appearance in the 1909 Carter car. In the 1930's General Motors introduced the Transitorq (ref. 4) and Austin produced the Hayes transmission (ref. 5). Neither of these toroidal traction transmissions were in service for very long. One factor that hindered the widespread acceptance of the earlier traction drives was the limited durability of roller component materials and the uncertainties in life prediction methods. High quality roller materials and lubricants are essential for long service life since high contact pressures often greater than 240 000 newtons per square centimeter (350 000 psi) maximum Hertz stress are needed to transmit significant power through these drives. Since the drive's design life is limited by roller fatigue which is a strong function of contact pressure, high powered traction drives have normally been restricted to limited-life applications (ref. 4). However, the power ratings and service life of modern traction drives have improved somewhat due to the production of cleaner bearing steels with better fatigue

resistance and the development of lubricants which possess better tractive properties (ref. 6).

Recently, an advanced toroidal traction transmission has been proposed for automotive application (ref. 7). Use of this CVT could greatly improve vehicle fuel economy but its acceptance depends heavily on conversion cost factors and its reliability for long term vehicle service.

Present methods to predict the design lives of traction elements are limited to empirical approximations based only on contact stress and rolling diameter (ref. 8). Because of the similarity in the failure mechanism between traction drive components and those experienced with rolling-element bearings, it is anticipated that the bearing fatigue life theory of Lundberg and Palmgren (ref. 9) can be successfully adapted to predicting traction drive life. In a similar vein, the work reported in reference 10 applied the statistical approach used by Lundberg and Palmgren for rolling-element bearings to life predictions for spur and helical gears. The surface fatigue life predictions from that analysis showed good agreement with actual test data.

In view of the aforementioned, the objective of the work reported herein is to (1) develop an analysis based on classical rolling-element bearing fatigue theory for predicting the design life of traction drive systems and (2) to demonstrate the use of this analysis by forecasting the fatigue life expectancy of an advanced toroidal traction transmission under a variety of operating conditions.

## SYMBOLS

a	semimajor axis, m (in.)
b	semiminor axis, m (in.)
c	orthogonal shear stress exponent
E	Young's modulus, $N/m^2$ (psi)
$\mathcal{E}$	elliptic integral of the first kind
e	Weibull exponent
F	curvature difference
$F_a$	axial force, N (lb)
$\mathcal{F}$	elliptic integral of the second kind
g	defined by eq. (18)
H	life, hr
h	depth to critical stress exponent

$K, K_1$	constants of proportionality
$k$	ellipticity ratio
$L$	life (millions of input shaft revolutions)
$l$	length of rolling track, m (in.)
$m$	drive ratio, (output speed/input speed)
$n$	speed, rpm
$Q_A$	actual roll body load, N (lb)
$Q_R$	required roll body load, N (lb)
$q$	maximum Hertz stress, $N/m^2$ (psi)
$R$	cam radius, m (in.)
$r$	radius, m (in.)
$T_{in}$	input torque, N-m (in.-lb)
$u$	stress cycles per revolution
$V$	stressed volume, $m^3$ (in. <sup>3</sup> )
$w$	semiwidth of rolling track, m (in.)
$X_c$	number of cavities
$X_r$	number of rollers per cavity
$z_o$	depth to maximum orthogonal reversing shear stress
$\alpha$	roller tilt angle, rad
$\beta$	preload cam angle, rad
$\theta$	roller cone angle, rad
$\mu$	traction coefficient
$\xi$	Poisson's ratio
$\rho$	curvature sum, $m^{-1}$ (in. <sup>-1</sup> )
$\tau_o$	maximum orthogonal reversing shear stress, $N/m^2$ (psi)
$\psi$	overload ratio

Subscripts:

$A, B$	elastic bodies
act	actual
c	cavity

d drive  
in input contact  
o output contact  
r roller  
req required  
t torus  
x,y reference planes

## ANALYSIS

### Fatigue Life Analysis

In 1947 Lundberg and Palmgren (ref. 9) published a theory for the failure distribution of ball and roller bearings. The mode of failure was assumed to be subsurface originated (SSO) fatigue pitting. Lundberg and Palmgren theorized that SSO fatigue pitting was due to high stresses in the neighborhood of a stress raising incongruity in the bearing material. The important parameters are number of stress cycles  $L$ , magnitude of the critical stress  $\tau_o$ , amount of volume stressed  $V$ , and depth below the surface at which the critical stress occurs  $z_o$ . The theory is widely used to predict rolling-element bearing fatigue life and was recently adapted for predicting the life of spur and helical gears (refs. 11 and 12). In reference 11 the formulation for life prediction of a steel gear set was confirmed with life data from full-scale spur gear tests. For a steel rolling element the number of stress cycles endured before failure occurs is given by the following equation (ref. 12):

$$L = \left( \frac{K_1 z_o^h}{\tau_o^c V} \right)^{1/e} \quad (1)$$

This equation is a modified form of the Lundberg-Palmgren theory for contact-fatigue life prediction and is applicable to gears, bearings, and other rolling-contact elements. For rolling-element bearings (and bodies in rolling contact in general) made of AISI 52100 steel, Rockwell C62 hardness, with bearing life at a 90 percent probability of survival, the following values are appropriate for use in equation (1) to determine life in millions of stress cycles (ref. 12):

$$\begin{aligned}
K_1 &= 1.428 \times 10^{95} \text{ (N and m units)} \\
&= 3.583 \times 10^{56} \text{ (lb and in. units)}
\end{aligned} \tag{2}$$

$$\begin{aligned}
e &= \frac{10}{9} \text{ (for elliptical shape point contact)} \\
&= \frac{3}{2} \text{ (for line contact)}
\end{aligned} \tag{3}$$

$$h = 2\frac{1}{3} \tag{4}$$

$$c = 10\frac{1}{3} \tag{5}$$

The stressed volume is given by

$$V = wz_0 l \tag{6}$$

where  $l$  is the length of the rolling track which is traversed during one revolution. The semiwidth of the rolling track is designated  $w$ , and  $z_0$  is the depth to the maximum orthogonal reversing shear stress.

If the body in question is subjected to several different contact loads during the load cycle, then Miner's rule (ref. 13) is useful in determining the element life as follows. Assume that loads  $Q_j$  ( $j = 1$  to  $k$ ) act during one load cycle of body  $i$ . First, each of the lives  $L_{ij}$  ( $j = 1$  to  $k$ ) is calculated by using equation (1) and assuming only the appropriate load  $j$  to be acting. Then the life of the  $i^{\text{th}}$  body when subjected to all  $k$  loads during a cycle is given by

$$L_i = \left( \sum_{j=1}^k \frac{1}{L_{ij}} \right)^{-1} \tag{7}$$

Heretofore the lives of the various elements were given in terms of millions of stress cycles. All bodies in the drive accumulate stress cycles at different rates because their speeds of rotation and number of stress cycles per revolution are not all the same. In order to compare lives of the various bodies clock time should be used. Assume that the speed in revolutions per minute of the  $i^{\text{th}}$  body is  $n_i$  and that there are  $u_i$  stress cycles per revolution. Then the life of body  $i$  in hours is given by



$$H_i = \frac{L_i}{u_i n_i} \left( \frac{10^6}{60} \right) \quad (8)$$

The life of the drive system is then found by applying Weibull's rule (ref. 15). If the drive system consists of  $n$  roll bodies and the life of each is designated  $H_i$  ( $i = 1$  to  $n$ ), then the drive life is given by

$$H_d = \left( \sum_{i=1}^n H_i^{-e} \right)^{-1/e} \quad (9)$$

### Contact Stress Analysis

The stress analysis of elastic bodies in contact was developed by H. Hertz (ref. 14). Hertz assumed homogeneous solid elastic bodies made of isotropic material which are characterized by Young's modulus  $E$  and Poisson's ratio  $\xi$ . Bodies A and B in contact are assumed to have quadratic surfaces in the neighborhood of the contact point.

Figure 1 shows two bodies in contact. Planes  $x$  and  $y$  are the respective planes of maximum and minimum relative curvature for the bodies. These planes, called the principal planes, are mutually perpendicular. Planes  $x$  and  $y$  must be chosen so that the relative curvature in plane  $x$  is greater than in plane  $y$ :

$$\frac{1}{r_{Ax}} + \frac{1}{r_{Bx}} > \frac{1}{r_{Ay}} + \frac{1}{r_{By}} \quad (10)$$

The radii of curvature may be positive or negative depending on whether the surfaces are convex or concave, respectively.

After the bodies are pressed together the contact point is assumed to flatten into a small area of contact which is bounded by an ellipse with major axis  $2a$  and minor axis  $2b$  as shown in figure 1. Plane  $y$  contains the major axis of the contact ellipse and plane  $x$  contains the minor axis. The ratio  $k = a/b$  is called the ellipticity ratio of the contact. The values of  $k$  range from 1 to  $\infty$  for various curvature combinations of contacting surfaces. For cylinders in contact, the ellipticity ratio is  $\infty$ , and the flattened area of contact is a rectangular strip. For spheres in contact the ellipticity ratio is 1. The first type is called line contact and all other types are called point contact. The theory of Hertz is summarized by Harris (ref. 15). The relation between ellipticity ratio and the geometry of the contacting bodies is given by the following transcendental equation:

$$F = \frac{\left(\frac{1}{r_{Ax}} + \frac{1}{r_{Bx}}\right) - \left(\frac{1}{r_{Ay}} + \frac{1}{r_{By}}\right)}{\rho} = \frac{(k^2 + 1) - 2\mathcal{F}}{(k^2 - 1)\mathcal{E}} \quad (11)$$

$$\rho = \frac{1}{r_{Ax}} + \frac{1}{r_{Bx}} + \frac{1}{r_{Ay}} + \frac{1}{r_{By}} \quad (12)$$

$$\mathcal{E} = \int_0^{\pi/2} \left[ 1 - \left(1 - \frac{1}{k^2}\right) \sin^2 \varphi \right]^{-1/2} d\varphi \quad (13)$$

$$\mathcal{F} = \int_0^{\pi/2} \left[ 1 - \left(1 - \frac{1}{k^2}\right) \sin^2 \varphi \right]^{1/2} d\varphi \quad (14)$$

where  $F$  is the curvature difference,  $\rho$  the curvature sum, and  $\mathcal{E}$  and  $\mathcal{F}$  are elliptic integrals of the first and second kind, respectively. Hamrock has developed an iterative scheme by which these equations may be solved (ref. 16). Hamrock's method was used to perform the stress calculations needed for this analysis.

The maximum surface contact pressure at the center of the ellipse is

$$q = \frac{3Q}{2\pi ab} \quad (15)$$

where

$$a = a^*g \quad (16)$$

$$b = b^*g \quad (17)$$

$$g = \sqrt[3]{\frac{3Q}{2\rho} \left( \frac{1 - \xi_A^2}{E_A} + \frac{1 - \xi_B^2}{E_B} \right)} \quad (18)$$

$$a^* = \sqrt[3]{\frac{2k^2\mathcal{E}}{\pi}} \quad (19)$$

$$b^* = \sqrt[3]{\frac{2\mathcal{E}}{\pi k}} \quad (20)$$

The maximum reversing orthogonal shear stress  $\tau_o$  occurs at a depth  $z_o$  underneath the surface of contact:

$$\tau_o = \frac{\sqrt{2t-1}}{2t(t+1)} q \quad (21)$$

$$z_o = \frac{1}{(t+1)\sqrt{2t-1}} b \quad (22)$$

where  $t$  is an auxiliary parameter which is related to  $k$  by

$$k = \left[ (t^2 - 1)(2t - 1) \right]^{-1/2} \quad (23)$$

Table I may be used to calculate the stresses and contact ellipse dimensions when it is impractical to solve equation (11).

#### Analysis of Toroidal Drive

A schematic diagram of the single cavity cone-roller toroidal (CRT) drive is shown in figure 2. This drive is of the same contact geometry as that presented in reference 7. The drive consists of one input and one output traction disk. The drive disk conforms to a toroidal shaped cavity with radius  $r_c$ . The radius of the torus is  $r_t$ . The output disk is driven by an input disk through the intermediate rollers, which are crowned with radius  $r_r$ . The intermediate roller is compressed between the drive disks by the pre-load cams in order to develop the contact loads necessary for transmitting the required power without gross slippage.

The cone rollers are offset from the cavity center by the amount  $r_c \cos \theta/2$ . The angle of tilt  $\alpha$  determines the drive ratio. For positive values of  $\alpha$  there is reduction in speed, and for negative values of  $\alpha$  the speed is increased through the drive. The equation relating drive ratio  $m$  to tilt angle  $\alpha$  is

$$m = \frac{r_{in}}{r_o} = \frac{r_t - r_c \cos\left(\frac{\theta}{2} - \alpha\right)}{r_t - r_c \cos\left(\frac{\theta}{2} + \alpha\right)} \quad (24)$$

Figure 3 shows a schematic of the dual cavity CRT drive. This dual cavity version is identical in contact geometry to the single cavity drive. Two cavities eliminate the need for large input and output thrust bearings. Notwithstanding the added size and complexity, it is probable that the dual cavity arrangement will be the first choice for an automotive application.

The traction drive is able to transmit power due to the resistance to slip that is developed in the thin film of oil in the rolling contact. The tractive force is a function of the type of oil, temperature, rolling and sliding speeds, degree of spin, surface finish, and normal contact force per unit width of contact. In order to prevent slipping at the traction contacts the normal load must be maintained above a certain minimum level so that the tractive force provided by the oil exceeds that required by the contact. This normal load is maintained by a rolling cam preload mechanism as shown in figure 4. The axial thrust  $F_a$  is related to the input torque  $T_{in}$  by

$$F_a = KT_{in} \quad (25)$$

where

$$K = \frac{1}{R \tan \beta} \quad (26)$$

The axial force then acts to pinch the cone roller and thus apply the necessary normal force. The actual normal force produced is given by

$$Q_{act} = \frac{F_a}{\sin\left(\frac{\theta}{2} - \alpha\right) X_c X_r} = \frac{KT_{in}}{\sin\left(\frac{\theta}{2} - \alpha\right) X_c X_r} \quad (27)$$

This normal force produced is only a function of the drive ratio, cam constant, and cone angle. However, the normal force that is required to assure sufficient traction is a function of all the factors previously mentioned that cause the formation of the EHD film at the disk-roller contact points. The traction coefficients can range from 0.03 to 0.08 (ref. 7). However, under the operating conditions which exist in the CRT, the practical upper limit is about 0.06. In the present analysis a constant available traction

coefficient of 0.06 is assumed at all conditions. This assumption allows the most generous relation between traction and compressive normal forces. Therefore the life analysis is optimistic for this type of drive.

The overload factor is the ratio of the actual normal force on the contact to the normal force required to transmit the given torque. The required contact force is given by

$$Q_{\text{req}} = \frac{T_{\text{in}}}{\mu r_{\text{in}} X_c X_r} \quad (28)$$

where

$$r_{\text{in}} = r_t - r_c \cos\left(\frac{\theta}{2} - \alpha\right) \quad (29)$$

From equations (27), (28), and (29) the expression for overload factor is obtained:

$$\psi = \frac{Q_{\text{act}}}{Q_{\text{req}}} = \frac{\mu}{\tan \beta} \left( \frac{r_t}{r} \right) \frac{\left[ 1 - \frac{r_c}{r_t} \cos\left(\frac{\theta}{2} - \alpha\right) \right]}{\sin\left(\frac{\theta}{2} - \alpha\right)} \quad (30)$$

The minimum overload occurs at roller tilt angle given by

$$\alpha^* = \frac{\theta}{2} - \cos^{-1}\left(\frac{r_c}{r_t}\right) \quad (31)$$

To provide some margin of safety against gross slipping, the minimum overload factor should be significantly larger than unity. In the present analysis a minimum overload factor of 1.2 was assumed.

The contact radii that are needed for the stress analysis may be found from figure 2. At the constant of input disk and roller the principal radii of curvature are

$$r_{A1} = \frac{r_{\text{in}}}{\cos\left(\frac{\theta}{2} - \alpha\right)} \quad (32)$$

$$r_{B1} = r_c \quad (33)$$

$$r_{A2} = -r_c \quad (34)$$

$$r_{B2} = r_r \quad (35)$$

For the contact of output disk and roller the radii are

$$r_{A1} = \frac{r_o}{\cos\left(\frac{\theta}{2} + \alpha\right)} \quad (36)$$

$$r_{B1} = r_c \quad (37)$$

$$r_{A2} = -r_c \quad (38)$$

$$r_{B2} = r_r \quad (39)$$

In order to determine the orientation of the contact ellipse in relation to the rolling track the inequality of equation (10) must be satisfied. Since it is not known a priori whether plane 1, the plane of rolling, or plane 2, the transverse plane, is the plane of the maximum curvature, the subscripts  $x$  and  $y$  were not used in the previous general equations.

If reference plane 1 is the plane of maximum curvature (plane  $x$ ) according to equation (1), then the major axis of the contact ellipse is perpendicular to the rolling track. Conversely, the minor axis of the contact ellipse is perpendicular to the rolling track if plane 2, the transverse plane, is the plane of maximum curvature (plane  $x$ ). For the dimensions and drive ratios used herein the plane of maximum curvature is indeed plane 1 and hence the major axis of the contact ellipse defines the width of the rolling track. This is true for both the input and output traction contacts.

The 90-percent life for each component of the traction drive is found for a given drive ratio by applying equations (1) and (7). The element lives given by equations (1) and (7) are in terms of stress cycles endured. The lives may also be expressed in hours of operation before pitting damage occurs by using equation (8). The input disk endures  $X_r$  stress cycles per revolution if there are  $X_r$  roller elements. The life of the input disk is given by

$$H_{in} = \frac{L_{in}}{X_r n_{in}} \left( \frac{10^6}{60} \right) \quad (40)$$

The life of the output disk is given by

$$H_o = \frac{L_o}{X_r n_o} \left( \frac{10^6}{60} \right) = \frac{L_o}{X_r n_i m} \left( \frac{10^6}{60} \right) \quad (41)$$

The intermediate roller elements contact both the input and output disks. For a unity drive ratio, the stress and contact ellipse dimensions are the same at both contacts. In this case the roller is subjected to two equal stress cycles per roller revolution.

When the drive ratio is not unity, the roller is subjected to two different states of contact stress per revolution. In this case, the life of the roller is denoted by  $(L_r)_{in}$  when the input is assumed to act alone on the roller. Similarly,  $(L_r)_o$  is the life if only the output contact load is acting. Then using Miner's rule (eq. (7)) the expected life of the roller is calculated:

$$L_r = \left[ \frac{1}{(L_r)_{in}} + \frac{1}{(L_r)_o} \right]^{-1} \quad (42)$$

In hours, the life of a single roller is determined by equation (8):

$$H_r = \frac{L_r}{n_r} \left( \frac{10^6}{60} \right) = \frac{L_r}{n_{in}} \frac{r_c \sin \frac{\theta}{2}}{r_{in}} \left( \frac{10^6}{60} \right) \quad (43)$$

The life of the single cavity drive consisting of the manifold of four elements (2 drive disks plus 2 rollers) is found according to Weibull's rule (eq. (9)):

$$H_d = \left[ \sum_{i=1}^4 \left( \frac{1}{H_i} \right)^e \right]^{-1/e} = \left[ \left( \frac{1}{H_{in}} \right)^e + \left( \frac{1}{H_o} \right)^e + 2 \left( \frac{1}{H_r} \right)^e \right]^{-1/e} \quad (44)$$

## RESULTS AND DISCUSSION

A single cavity CRT drive was analyzed for life and Hertz contact stress. The dimensions of the drive were taken equal to those of the drive in reference 7, and are as follows:

Torus diameter, cm (in.) . . . . .	11.43 (4.50)
Cavity diameter, cm (in.) . . . . .	8.84 (3.48)
Roller crown radius, cm (in.) . . . . .	3.12 (1.23)

Figure 5 gives the overload factor for the drive as a function of ratio from equation (29). The minimum overload factor occurs at drive ratio 0.647. This minimum overload factor was set equal to 1.2 to provide a 20 percent margin of safety against slip. The variation in the overload factor with ratio is primarily dependent on traction drive geometry. If the overload factor becomes too large then the fatigue life of the drive will be penalized. The cone roller toroidal drive under study experiences significantly less variation in the overload factor than other comparable toroidal roller drives (ref. 7). For the case presented, the overload factor ranged from 1.2 to 1.8.

### Parametric Variations

Figure 6 summarizes the results of the stress analysis. The analysis showed the Hertz stress at each traction zone is proportional to the cube root of input torque for a constant drive ratio. If the torque is held constant, there is a 2.3 to 1 variation in the Hertz stress at the input disk-roller contact as the drive ratio increases from 1:3 to 4:1. The larger maximum Hertz stress is on the input contact for drive ratios smaller than 1:1 and on the output contact for higher ratios.

Figure 7 shows the load-life curves that resulted from the analysis. Three different ratios were considered. The curves presented show that for any fixed operating ratio the life is related to load by

$$L \propto (T_{in})^{-3}$$

The most severe service condition for any given load is the 1:3 reduction ratio.

The lives of the individual components were also calculated. The relative lives of the drive disks and intermediate roller are given in table II. The lives are normalized by the drive system life for each ratio considered. For low ratio the life limiting component is the input disk and for the high ratio the life is limited by the output disk. If the component lives are normalized by the system life at 1:3 drive ratio, then the effect of ratio on each component life is included. Those results are in table III.

Table IV is a summary of the effect that geometry changes have on life and Hertz stress. Life and stress calculations were performed for scaled up versions of the basic single cavity CRT drive with two rollers. Also, the effect of adding more rollers, as well as one more cavity, was investigated. It was found that by a size increase of 10 percent a life equivalent to the three-roller drive (case 5) was obtained. To achieve life par-



ity with the double cavity drive the basic drive size had to be increased by about 20 per cent. The drive life increases rapidly with size according to the proportionality

$$L \propto (\text{size factor})^{8.4}$$

Therefore, if the costs of manufacture are considered it is more cost effective to simply increase size in order to achieve higher load ratings. However, with increased sizes the rolling speed at the traction contacts increases and the contact stress decreases. It is known that the traction coefficient decreases if rolling speed increases or contact stress decreases (ref. 17). Therefore, due consideration must be given to providing sufficient preload to compensate for the decreased traction coefficient that is available. A derating of the life expectation will be the final result of such a compensation adjustment.

### Design Study

A design study similar to that presented in reference 7 was also performed. The double cavity drive (case 6) was incorporated into the vehicle drive train as shown in figure 8. This drive system is modeled after the system described in the aforementioned reference which used the 4-liter (250 CID) six-cylinder engine and Plymouth B body. For the range of driving conditions considered, the torque input to the toroidal drive ranged up to 121 newton-meters (1071 in.-lb), which corresponds to an engine output of 102 horsepower at 4000 rpm. Figure 9 shows the calculated life as a function of drive ratio. Figure 10 shows the calculated values of maximum Hertz stress as a function of drive ratio.

For combinations of low ratio and high torque due to sudden overloads, the maximum Hertz stress on the input contact can exceed the AFBMA brinell limit of 517 000 newtons per square centimeter (750 000 psi). The brinell limit is defined as that maximum Hertz stress which would produce indentations of 0.0001 centimeter per centimeter of roller diameter of AISI 52100 steel of Rockwell C64 hardness (ref. 18). In reference 19, figure 7 also shows that a Hertz stress of 517 000 newtons per square centimeter (750 000 psi) will cause an indentation of 0.0001 times the ball diameter. However, the elastic limit of the material is exceeded at much lower stresses, and the 0.0001 times diameter criterion may not apply as a definition of excessive deformation for a toroidal drive. This is because the rolling elements in bearings generally run over the same track; therefore, a slight amount of grooving would have little adverse effect. However, since the intermediate rollers in a toroidal drive are required to sweep over the surface of the input and output disks in order to change ratio, even a small amount of grooving can have a detrimental effect on roller component life and the ability to adjust

drive ratio. In reference 15, equations for plastic deformation of ball bearings are given in chapter 11. Using those equations, stresses of only 345 000 to 414 000 newtons per square centimeter (500 000 to 600 000 psi) are required to cause a deformation of 0.0001 times the ball diameter.

In order to make the results of the analysis more useful for design assessment, calculated values of life and maximum Hertz stress are plotted against vehicle speed in figure 11. The life is given in hours for three different engine conditions of power output and speed. Also shown as dashed lines are the tire slip limit and the zero grade - zero wind road load conditions which span the probable values of vehicle operating parameters. If a representative vehicle operating point of 48 kilometers per hour (30 mph), 29 engine horsepower at 2000 rpm, and 1.2 drive ratio is selected, the resulting theoretical life is 331 hours at a 90-percent probability of survival. This condition can be taken as an average approximation of the operating parameters over the service life of the vehicle. If the vehicle is operated continuously at the tire slip limit, the drive life would be just under 10 hours.

### Life Adjustment Factors

In recent years, improvements in rolling-element bearing analysis, materials, processing, and manufacturing techniques have generally permitted an increase in the expected life of a bearing contact for a given application. In reference 20 bearing life adjustment factors have been developed which, when applied to Lundberg-Palmgren fatigue life predictions, will provide much better estimates of bearing fatigue performance. Many of these factors are also applicable to toroidal drive life predictions. The factors considered are (1) materials factor, (2) processing factor, and (3) lubrication factor.

It is anticipated that the contacting elements of the CRT drive will be made from consumable vacuum melted (CVM) AISI 52100 steel which has become the bearing industry's standard material. According to reference 20, the use of CVM processed AISI 52100 steel in a bearing application would result in a life amplification factor of 6 to be applied to theoretical life estimates.

With regard to traction lubricant fatigue life effects, some investigators have suggested that certain types of traction lubricants (viz, synthetic cycloaliphatic fluids) may also show superior fatigue life performance when compared to conventional lubricants under identical operating conditions (ref. 6). Unfortunately, the life improvement potential of these types of traction oils is somewhat in dispute in view of the research of reference 21, which could not establish any statistically significant fatigue life advantages. At this time there is no firm basis for assigning life improvement factors based on the traction lubricants' chemical effects.

However, it is well known that the thickness of lubricant film which separates contacting machine elements can have a strong influence on the contacts' fatigue life. In reference 20, the effectiveness of a lubricant film in terms of the ratio of film thickness to composite surface roughness  $h/\sigma$  is related to fatigue life. The lubricant-fatigue factor has been assigned a value of 1.0 when  $h/\sigma$  has a value of approximately 1.3.

In the case of the CRT drive for the representative operating point of 29 horsepower, 48.3 kilometers per hour (30 mph), and a drive ratio of 1.2, the calculated film thicknesses of the input and output contacts are 0.51 and 0.48 micrometer (20 and 19  $\mu$ in.), respectively, using the Archard and Cowking film formula (ref. 20). This calculation, the details of which appear in the appendix utilizes an advanced commercially available traction fluid with a contact inlet temperature of 344 K (160° F). These film thickness values must be regarded as quite marginal considering that they were calculated for a rather modest operating condition and that a surface finish of 0.61 micrometer (24  $\mu$ in.) rms has been proposed for the CRT drive elements (ref. 7). In fact, the calculated input and output  $h/\sigma$  values of 0.59 and 0.56, respectively, are so low at this condition that it is questionable whether long-term operation can be sustained without drastic surface distress. If operation is assumed possible, an 80-percent reduction in fatigue life would be expected (ref. 20). Thus, the sixfold fatigue life increase due to the CVM 52100 steel is offset by a lubrication factor of 0.2 for a net 1.2 life improvement factor.

In summary, the expected life at the selected duty cycle point based on classical bearing fatigue theory and the experience factors mentioned is 397 hours ( $1.2 \times 331$ ).

The expected life reported in reference 7 for the identical conditions is approximately 38 times longer. The mileage life using the methods reported herein is approximately 19 200 kilometers (11 900 miles) for this representative condition. Based on the life-scaling relations presented earlier, the CRT drive size should be scaled up to approximately 130 percent of its current size to provide for 160 900 kilometers (100 000 miles) of service. This approach is acceptable if the anticipated loss in the available traction coefficient due to increased surface speeds and lower contact pressures is tolerable.

It should be pointed out that at operating conditions which are more stringent than the condition examined, insufficient lubricant film will be generated to separate the relatively rough 0.61 micron (24  $\mu$ in.) rms roller surfaces and gross surface damage would most likely result. To combat loss of lubricant film the speed of the contact should be increased, the temperature of the lubricant should be lowered, and the surface finish of the roller elements should be greatly improved. These improvements which will undoubtedly add to the cost and possibly the size of the drive are necessary for reliable operation.

## SUMMARY OF RESULTS

A contact fatigue life analysis for traction drives has been developed. The life analysis was based on a modification of Lundberg-Palmgren theory which is the basis of rating bearing life in the rolling-element bearing industry.

The fatigue life analysis was used to do a parametric study of a toroidal type traction drive. The design study was compared to previously published work for the dual cavity toroidal drive as applied to a typical compact passenger vehicle drive train. Drive life and contact stresses were computed using a computer program based on the method presented herein. A constant traction coefficient of 0.06 was selected and a minimum contact overload factor of 1.2 to preclude slip was used. Conditions of elasto-hydrodynamic lubrication of the contacts were computed at a selected operating point and life adjustment factors due to advances in materials and lubricants technology were considered.

The following results were obtained from a study of a toroidal drive with a torus diameter of 11.43 centimeters (4.50 in.), a cavity diameter of 8.84 centimeters (3.48 in.), and a roller crown radius of 3.12 centimeters (1.23 in.):

1. For a representative duty cycle condition wherein the engine delivers 29 horsepower at 2000 rpm with the vehicle moving at 48.3 kilometers per hour (30 mph) at a drive ratio of 1.2, the life of the CRT drive is 19 200 kilometers (11 900 mile). This is 38 times less than previously stated in the literature for the same conditions.

2. For the previously stated conditions and a surface finish of 0.61 micrometer (24  $\mu$ in.) rms, the ratio of lubricant film thickness to composite surface roughness was less than 0.6. The lubricant film is barely sufficient to prevent severe roller component surface distress.

3. For conditions of high reduction ratio and high transmitted loads, the brinelling limit of high quality, hardened bearing steel is approach for the input disk-roller contact. Shock or dynamic overloading may cause significant brinelling of the input disk or roller.

4. Life is directly proportional to the 8.4 power of size. To provide for 160 900 kilometers (100 000 mile) of vehicle service life at the aforementioned duty point the CRT drive size must be scaled to approximately 130 percent of its current size. This can be accomplished only if the loss in available traction coefficient due to the increased surface speeds and reduced contact pressures is not too severe.

5. Drive life is inversely proportional to the cube of torque.
6. Hertz stress is proportional to cube root of input torque.

Lewis Research Center,  
National Aeronautics and Space Administration,  
and  
U.S. Army Air Mobility R&D Laboratory,  
Cleveland, Ohio, July 9, 1976,  
505-04.

## APPENDIX - ELASTOHYDRODYNAMIC FILM THICKNESS CALCULATION

The following lubricant film thickness equation for elliptical contacts was developed by Archard and Cowking (as reported in ref. 20):

$$h = 2.04 \left(1 + \frac{2R_1}{3R_2}\right)^{-0.74} \left[ \frac{\mu_0 \alpha (u_1 + u_2)}{2} \right]^{0.74} R_1^{0.407} \left[ \frac{E}{(1 - \xi^2)Q} \right]^{0.074} \quad (A1)$$

where

- h film thickness, m (in.)
- $u_1, u_2$  surface velocities of rolling elements, m/sec (in./sec)
- Q contact load between rolling elements, N (lb)
- $R_1, R_2$  equivalent contact radii in orthogonal directions, 1 refers to direction of rolling, m (in.)
- E Young's modulus of elasticity, N/m<sup>2</sup> (psi)
- $\xi$  Poisson's ratio
- $\mu_0$  lubricant absolute viscosity at atmospheric pressure, N-sec/m<sup>2</sup> (lb-sec/in.<sup>2</sup>)
- $\alpha$  reciprocal asymptotic isoviscous pressure-viscosity coefficient, (N/m<sup>2</sup>)<sup>-1</sup>, (psi<sup>-1</sup>)

The equivalent radius of contacting bodies in the plane of rotation is given by

$$R_1 = \left( r_{A_1}^{-1} + r_{B_1}^{-1} \right)^{-1} \quad (A2)$$

and in the plane transverse to the direction of rolling by

$$R_2 = \left( r_{A_2}^{-1} + r_{B_2}^{-1} \right)^{-1} \quad (A3)$$

where  $r_{A_1}$ ,  $r_{A_2}$ ,  $r_{B_1}$ , and  $r_{B_2}$  are the principal radii of curvature as defined before.

At the representative duty cycle point the following conditions exist:

Vehicle speed, km/hr (mph) . . . . .	48.78 (30)
Engine power, hp . . . . .	29
Engine speed, rpm . . . . .	2000
Drive input speed, rpm . . . . .	3000
Drive ratio . . . . .	1.1653
Contact load between rolling elements, N (lb) . . . . .	11 782 (2648.8)
Surface velocities of rolling elements, $u_1 = u_2$ , m/sec (in./sec) . . . . .	9.739 (383.43)
Young's modulus of elasticity, E, N/m <sup>2</sup> (psi) . . . . .	$2.07 \times 10^{11}$ ( $30 \times 10^6$ )
Poisson's ratio, $\xi$ . . . . .	0.3
Equivalent contact radii in orthogonal directions, m (in.):	
$R_1$ (input) . . . . .	$2.397 \times 10^{-2}$ (0.9438)
$R_2$ (input) . . . . .	$10.66 \times 10^{-2}$ (4.1965)
$R_1$ (output) . . . . .	$2.057 \times 10^{-2}$ (0.8099)
$R_2$ (output) . . . . .	$10.66 \times 10^{-2}$ (4.1965)

For a synthetic cycloaliphatic traction fluid at a temperature of 344 K (160° F),

$$\mu = 0.010 \text{ N-sec/m}^2 (1.452 \times 10^{-6} \text{ (lb-sec/in.}^2\text{)}) \text{ (ref. 21)} \quad (\text{A4})$$

$$\alpha = 2.06 \times 10^{-8} \text{ m}^2/\text{N} (1.42 \times 10^{-4} \text{ psi}^{-1}) \text{ (ref. 22)} \quad (\text{A5})$$

The following film thicknesses were calculated for the input and output contacts:

$$h_{\text{input}} = 51 \text{ } \mu\text{m} (20 \times 10^{-6} \text{ in.}) \quad (\text{A6})$$

$$h_{\text{output}} = 48 \text{ } \mu\text{m} (19 \times 10^{-6} \text{ in.}) \quad (\text{A7})$$

Usually the root mean square (rms) surface finishes of the contacting bodies  $\sigma_1$  and  $\sigma_2$  are used to determine the composite surface roughness as follows:

$$\sigma = \left( \sigma_1^2 + \sigma_2^2 \right)^{1/2} \quad (\text{A8})$$

In reference 7 the proposed roller surface finishes are 0.61 micrometer (24  $\mu$ in. rms); thus,  $\sigma = 0.86$  micrometer (33.9  $\mu$ in.) and the  $h/\sigma$  ratios for the input and output are

$$(h/\sigma)_{\text{input}} = 0.59 \quad (\text{A9})$$

$$(h/\sigma)_{\text{output}} = 0.56 \quad (\text{A10})$$

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TABLE I. - VALUES OF CONTACT

## STRESS PARAMETERS

F	a*	b*	t
0	1	1	1.2808
.1075	1.0760	.9318	1.2302
.3204	1.2623	.8114	1.1483
.4795	1.4556	.7278	1.0993
.5916	1.6440	.6687	1.0701
.6716	1.8258	.6245	1.0517
.7332	2.011	.5881	1.0389
.7948	2.265	.5480	1.0274
.83495	2.494	.5186	1.0206
.87366	2.800	.4863	1.0146
.90999	3.233	.4499	1.00946
.93657	3.738	.4166	1.00612
.95738	4.395	.3830	1.00376
.97290	5.267	.3490	1.00218
.983797	6.448	.3150	1.00119
.990902	8.062	.2814	1.000608
.995112	10.222	.2497	1.000298
.997300	12.789	.2232	1.000152
.9981847	14.839	.2070	1.000097
.9989156	17.974	.18822	1.000055
.9994785	23.55	.16442	1.000024
.9998527	37.38	.13050	1.000006
1	$\infty$	0.	1.000000

TABLE II. - COMPONENT LIVES NORMALIZED BY

## DRIVE SYSTEMS LIFE AT VARIOUS RATIOS

## AND CONSTANT LOAD

Ratio	Drive system	Input disk	Output disk	Single roller
1:3	1	1.75	371	3.75
1:1	1	3.45	3.45	3.51
4:1	1	1498	1.73	3.78

TABLE III. - COMPONENT LIVES NORMALIZED BY  
DRIVE SYSTEM LIFE AT CONSTANT  
LOAD AND 1:3 RATIO

Ratio	Drive system	Input disk	Output disk	Single roller
1:3	1	1.75	371	3.75
1:1	43.6	151	151	153
4:1	2.84	4253	4.92	10.73

TABLE IV. - EFFECT OF SIZE, NUMBER OF ROLLERS, AND  
NUMBER OF CAVITY BANKS ON CONTACT FATIGUE  
LIFE AND HERTZ STRESS

Drive configuration	For any fixed ratio and load	
	Life increase, percent	Reduction in Hertz stress, percent
1. Basic CRT drive: 11.43 cm (4.50 in.) torus diameter 8.84 cm (3.48 in.) cavity diameter 3.12 cm (1.23 in.) roller crown radius single cavity with 2 rollers	~	~
2. 10 Percent larger	123	9
3. 20 Percent larger	363	17
4. Double size	34 000	50
5. Same as 1 except 3 rollers	129	13
6. Same as 1 except 2 cavities with 2 rollers per cavity	329	21

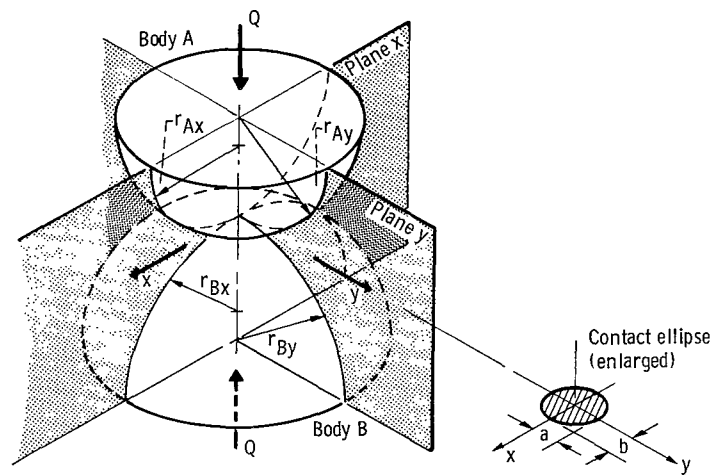


Figure 1. - Geometry of contacting solid elastic bodies.

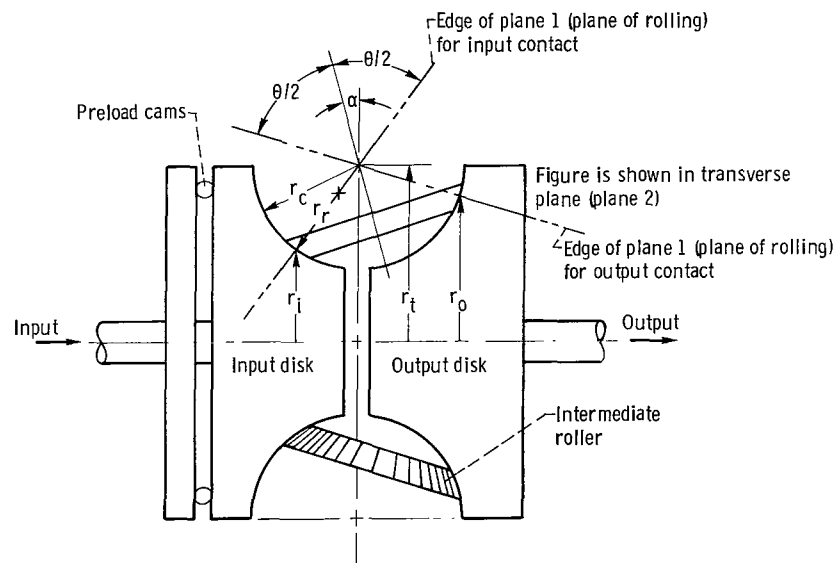


Figure 2. - Single cavity cone-roller toroidal drive. Ratio continuously variable by tilting plane of rollers. Tilt designated by angle  $\alpha$ . Drive rollers shown in speed reducer position.

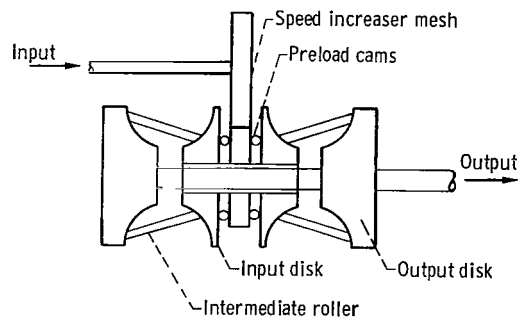


Figure 3. - Dual cavity cone roller toroidal drive.

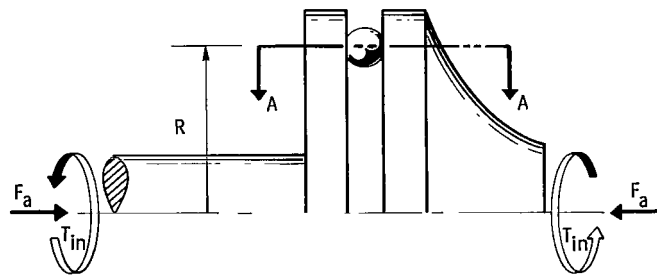
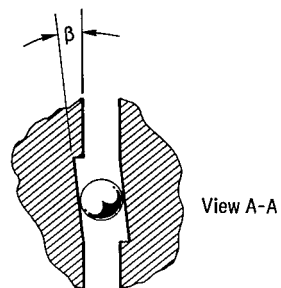


Figure 4. - Preload mechanism using rolling cams.

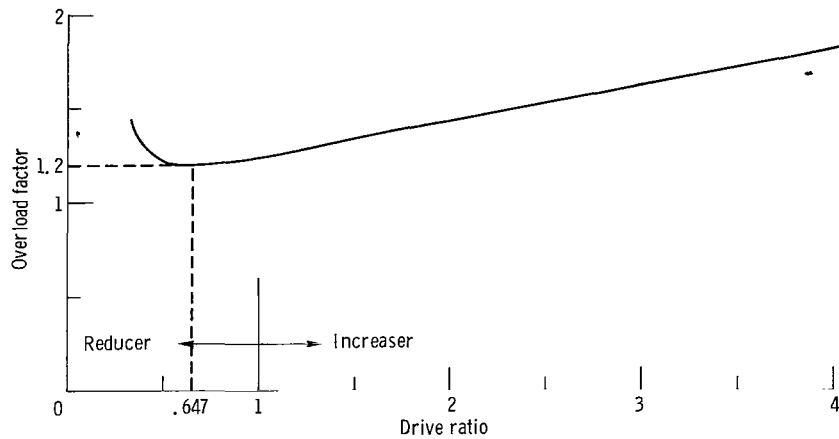


Figure 5. - Overload factor as function of drive ratio.

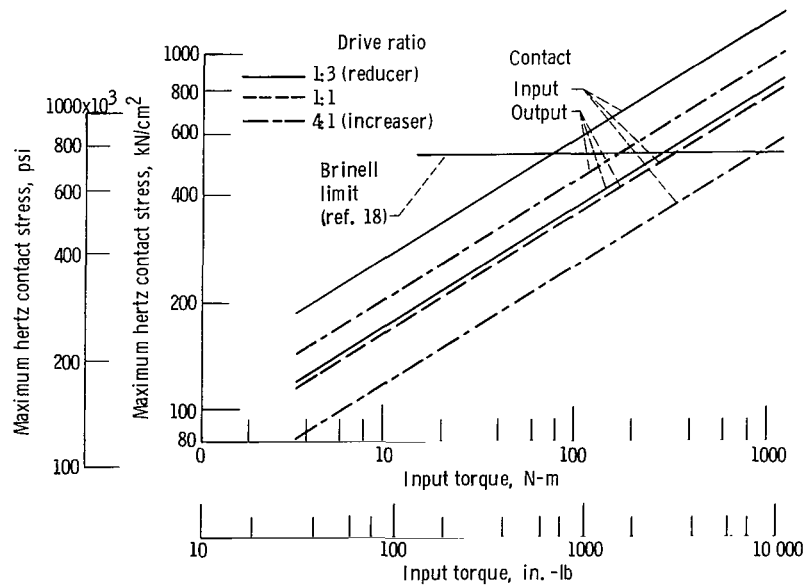


Figure 6. - Maximum Hertz stress at input and output contacts as function of input torque and drive ratio. Case I - single cavity toroidal drive with two rollers.

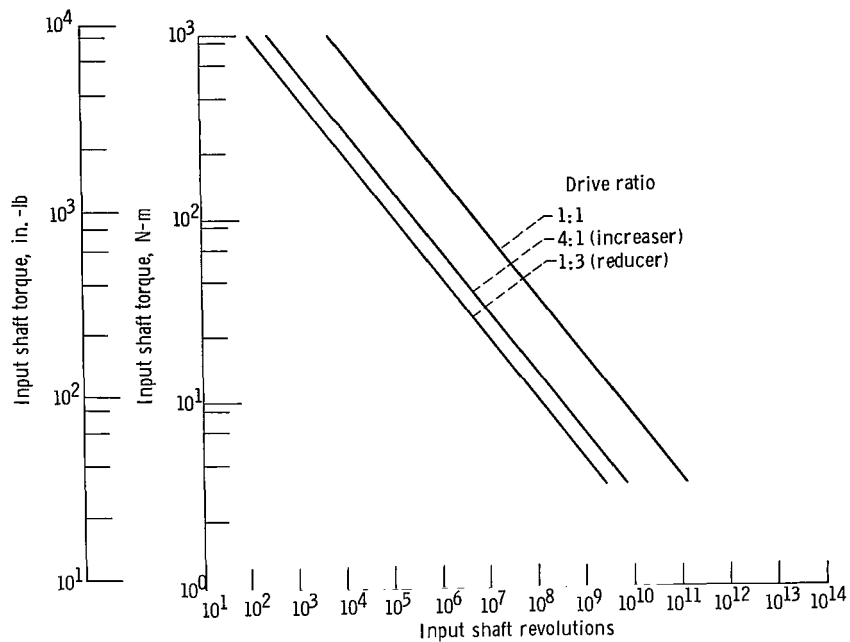


Figure 7. - Load-life curves for toroidal traction drive. Single cavity with two rollers.

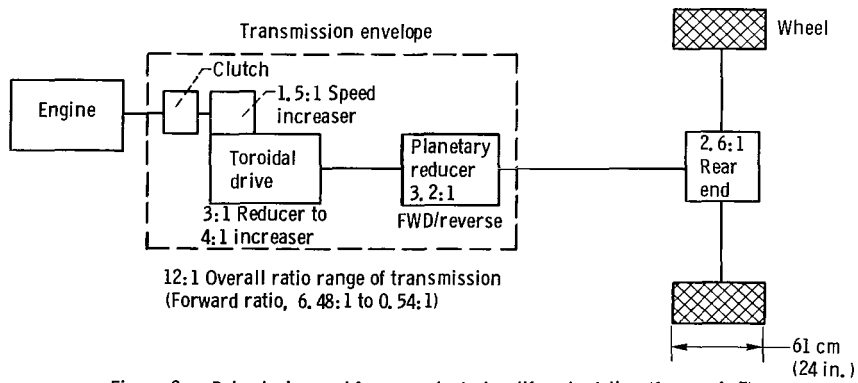


Figure 8. - Drive train used for example design life calculation (from ref. 7).

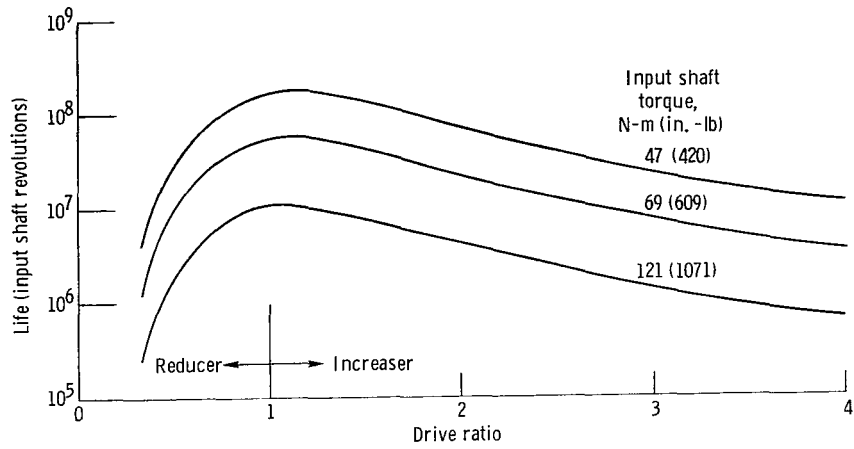


Figure 9. - Drive life as function of drive ratio for dual cavity CRT drive with two rollers per cavity.

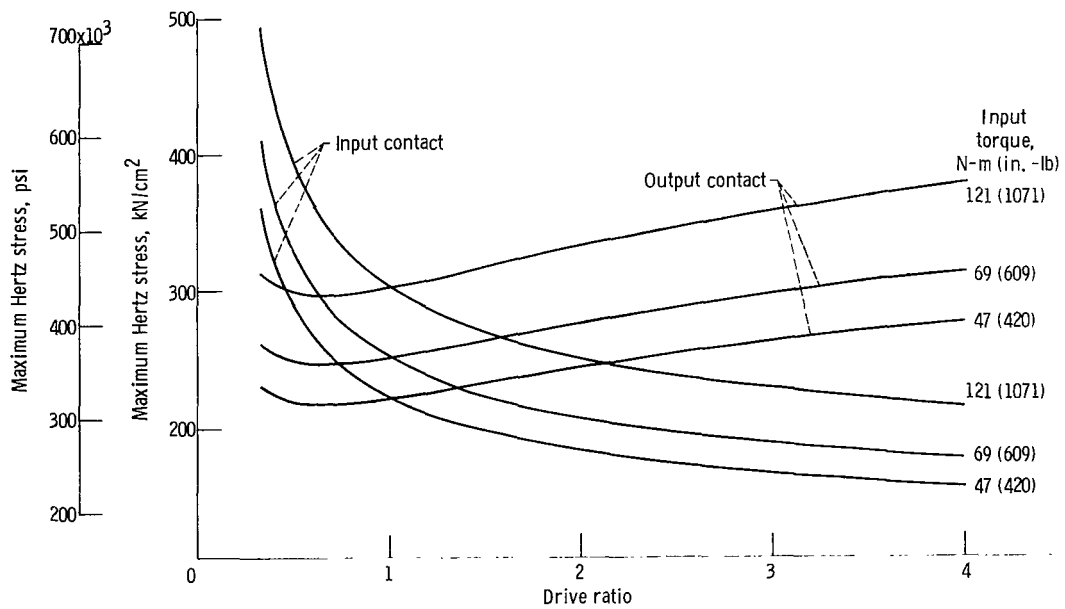


Figure 10. - Maximum Hertz stress at input and output contacts as function of ratio and input torque. Dual cavity drive (case 6).



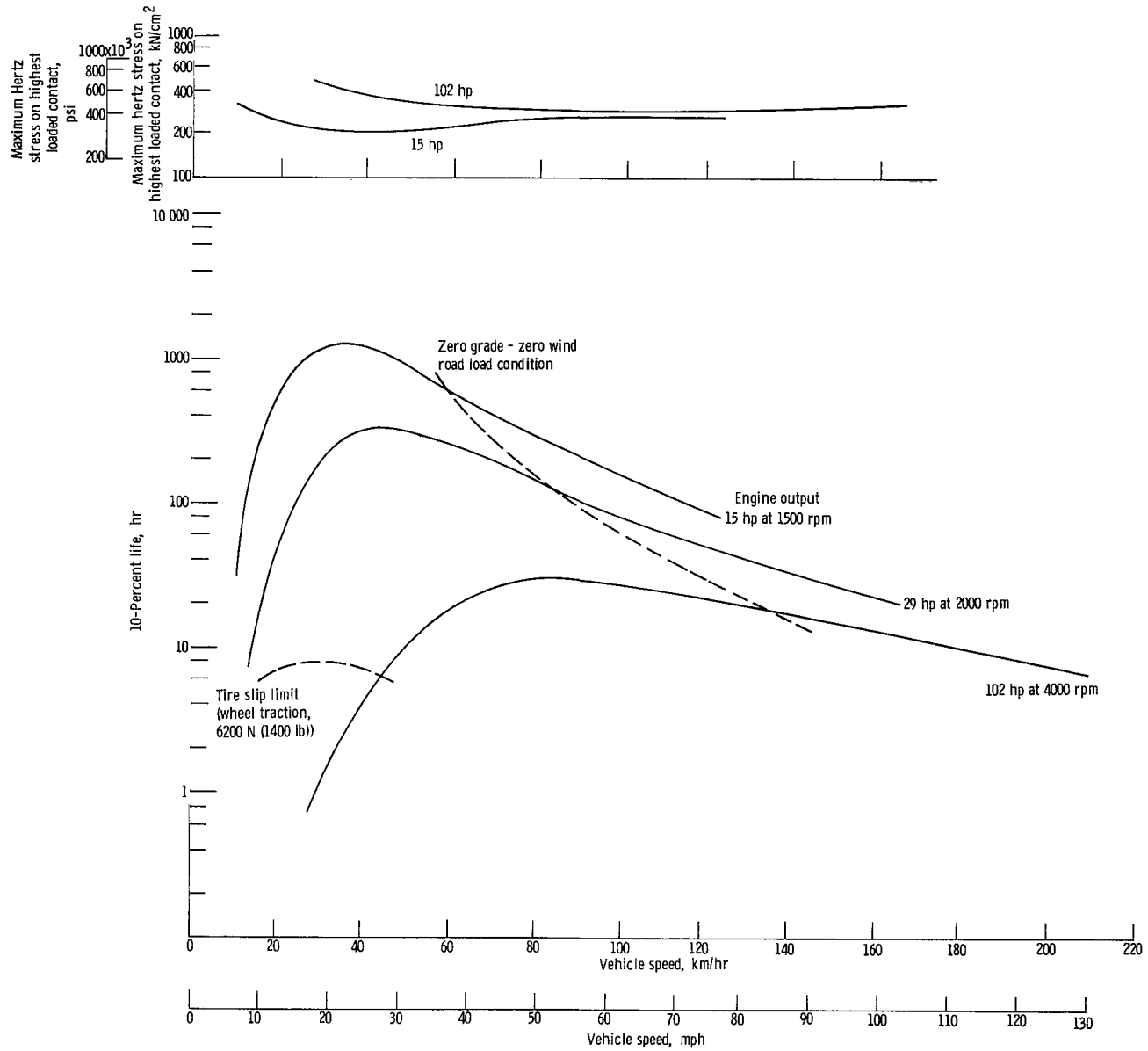


Figure 11. - Drive life and maximum Hertz stress as function of vehicle speed and transmitted power. Dual cavity drive



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